Topology

B. Math. II

Mid-Term Examination

Instructions: All questions carry equal marks.

1. Let S_Ω be the minimal uncountable well-ordered set. Let X_0 be the subset consisting of all elements x such that x has no immediate predecessor. Prove that X_0 is uncountable.

2. Let $\mathbb{R}$ denote the set of real numbers. Prove that the dictionary order topology on the set $\mathbb{R} \times \mathbb{R}$ is the same as the product topology $\mathbb{R}_d \times \mathbb{R}_{std}$, where $\mathbb{R}_d$ denotes $\mathbb{R}$ with discrete topology. Compare this topology with the standard topology on $\mathbb{R}^2$.

3. Let Y be an ordered set. Let X be a topological space and let $f : X \to Y$ be a continuous map with respect to the ordered topology on Y. Prove that the map
$$h(x) = \min\{f(x), g(x)\}$$
is a continuous map from X to Y.

4. Let $p : X \to Y$ be a quotient map such that $p^{-1}(y)$ is connected for every $y \in Y$. Prove that X is connected if and only if Y is connected.

5. Define **linear continuum**. If X is an ordered set in the order topology, prove that X is connected if and only if X is a linear continuum.

6. Prove that finite product of compact spaces is compact.